Shape Optimization and the (Generalized) Total Variation of the Normal as Prior for Geometrically Inverse Problems

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Abstract: The focus of this talk is shape optimization, i.e., optimal control problems subject to partial differential equations, where the domain is the unknown to be found. Geometrically inverse problems, such as finding the optimal emitter shape in acoustics, reconstructing material inclusions or constructing airfoils with desired flow characteristics are all examples of geometrically inverse problems. Of particular note here are also geo-electrical impedance tomography problems.

Being ill-posed, finding the desired solution depends on using a proper regularization technique. As such, priors for shape optimization are discussed, which foster the detection of non-smooth objects. To this end, we propose to use the total variation (TV) of the outer normal as prior. Due to the typical sparsity behavior of $L_1$ or TV-regularization terms, we expect the creation of piecewise flat objects. It turns out that this prior behaves quite differently in a continuous and discrete setting and we study critical shapes, i.e., those shapes, which have zero directional derivatives, in each case.

Finally, we discuss optimization algorithms to numerically solve the resulting non-smooth optimization problems. To this end, we introduce ADMM-type methods on surfaces and manifolds, with the split Bregman approach being our method of choice.

(joint work with Marc Herrmann, Lukas Baumgärtner, Roland Herzog, Ronny Bergmann and Jose Vidal Nuñez)